Quiz #5, MATH 10A, Section 101 & 105 — 10/10

Name: FRANCO

1) Find the general F with $F'(x) = x^{-5} - \frac{3}{x} + \cos(2\pi x)$ $(x > 0)$ (2 points)

2) Find $F(x)$ such that $F'(x) = x^{4/3} + e^{e^x} \cdot e^x$, $F(1) = 5 + e^e$ (3 points)

3) Calculate the area between the graph of the curve $y = x^3 - 2$ and the x-axis from $x = 0$ to $x = 2$ (3 points)

1) $F(x) = \left(\frac{x^{-4}}{-4}\right) - 3\left(\ln(x)\right) + \frac{\sin(2\pi x)}{2\pi} + C$

$= -\frac{x^{-4}}{4} - 3\ln(x) + \frac{\sin(2\pi x)}{2\pi} + C$

2) Observe that for $\underbrace{e^{e^x}}_{} \cdot \underbrace{e^x}_{}$

Reverse chain rule: $g'(f(x))\, f'(x)$ $\quad g_{(x)} = e^x$, $f(x) = e^x$

$\Rightarrow e^{e^x} \cdot e^x = \left(e^{e^x}\right)'$

Substitution $u = e^x$, $u' = e^x$

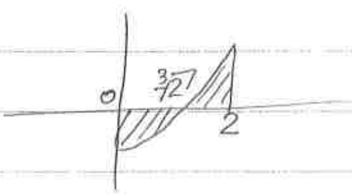
$\Rightarrow e^{e^x} \cdot e^x = e^u \cdot u' = (e^u)' = \left(e^{e^x}\right)'$

$\Rightarrow F(x) = \frac{x^{7/3}}{7/3} + e^{e^x} + C = \frac{3}{7}x^{7/3} + e^{e^x} + C$

$5 + e^e = F(1) = \frac{3}{7} \cdot 1^{7/3} + e^{e^1} + C = \frac{3}{7} + e^e + C \Rightarrow C = \frac{32}{7}$

$F(x) = \frac{3}{7}x^{7/3} + e^{e^x} + \frac{32}{7}$

3) Since $x^3 - 2$ is ≤ 0 for $x \in [0, \sqrt[3]{2}]$ and > 0 for $[\sqrt[3]{2}, 2]$



$\text{Area} = -\int_0^{\sqrt[3]{2}} (x^3 - 2)\,dx + \int_{\sqrt[3]{2}}^{2} (x^3 - 2)\,dx$

$= \left(-\frac{x^4}{4} + 2x\right)\Big|_0^{\sqrt[3]{2}} + \left(\frac{x^4}{4} - 2x\right)\Big|_{\sqrt[3]{2}}^{2}$

$= -\frac{\sqrt[3]{16}}{4} + 2\sqrt[3]{2} + \left(\frac{16}{4} - 2(2) - \frac{\sqrt[3]{16}}{4} + 2\sqrt[3]{2}\right)$

$= 4\sqrt[3]{2} - \frac{\sqrt[3]{2} \cdot 2}{2} = 3\sqrt[3]{2}$